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Stabilization of Continuous-time and Discrete-time Switched Systems: A Review

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Abstract : *Stability problem for a class of linear continuous time and discrete time systems is well understood since long. This analysis has also been extended to switched linear systems with continuous and discrete descriptions. For such cases, arbitrary switching problem is addressed by constructing common quadratic and non-quadratic Lyapunov function. Moreover, keeping in view the invertible time delays in dynamical systems due to internal factors or external environment, study of switched systems with delays has become quite challenging. The present work, highlights the state of art review of switched systems and underlying methodologies to ensure stabilization of such system with individual systems having stable or unstable dynamics. Further, current status and future directions of possible scope and open challenges in this area are also highlighted for completeness.*

Keywords: *hybrid systems, dwell-time, Lyapunov functions, stability, switched systems*

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I. Introduction

Stability analysis and control of linear continuous time and discrete time systems have been studied since long by various researchers. This analysis has also been extended to hybrid systems which are commonly used in practice. Switched control systems find prominent place in linear continuous and discrete time systems class. Many contributions have been made in literature to address the issues of stability and controller design for linear switched systems over the years. In this paper, the review of stabilization, controllability and observability of switched hybrid systems is presented. The objective of this paper is to highlight the milestones in the development of the aforementioned areas and to review the problems that remain open. Firstly, various tools for stability analysis, including common Lyapunov and multiple Lyapunov functions, LMIs and LaSalle's invariance principle etc. are discussed and brief description of pioneering works of some authors in this direction is also made. Secondly, the controllability concept in switched hybrid control systems is studied.

Various related definitions of controllability are introduced and brief summary about the most recent and early papers written in this domain is presented. Along with that, work on observability and related concepts is also highlighted and finally, the problems arising during the analysis of switched hybrid systems are stated. The paper is concluded with a note on future directions.

II. Stabilization of hybrid switched systems

A switched system is a hybrid dynamical system that consists of either discrete-time or continuous time subsystems and a rule that regulates the switching among them [1]. Continuous-time switched nonlinear systems can be modeled as:

$$\dot{x}(t) = f_i(x(t), u(t)),\ t \in R^+,\ i \in I = \{1,.., N\} \tag{1}$$

where x denotes the states, $\mathbf{x} \in R^n$, the control $\mathbf{u} \in R^m$, R^+ represents non-negative real numbers, the finite set I denotes an index set and represents the collection of discrete modes. In this way, a class of discrete-time switched systems can be represented as a collection of difference equations:

$$x[k+1] = f_i(x[k], u[k]);\ k \in Z^+,\ i \in I = \{1,.., N\} \tag{2}$$

where Z^+ represents non-negative numbers.

A collection of 'm' switched systems consists of m individually unstable linear and autonomous systems. The switching sequence decides the dynamics of the switched system by specifying the point and time of switching [2]. The investigation of asymptotic stabilization of m switched systems through Lyapunov functions was initiated by Peleties and Marlo. Desoer in 1960s and Branicky in 1994 made the observation, to constrain rate of switching, Lyapunov functions could be used to derive laws in a way to ensure stability [3].

Liberzon showed that necessary and sufficient condition for asymptotical stability of switched systems was existence of a common Lyapunov function for all subsystems. Numerous techniques to construct this Lyapunov function were propounded by Decarlo, Branicky, Lennartson, Liberzon and Morse a few decades back. However, the aforementioned concept was not of much avail as finding a common Lyapunov function was considered difficult or sometimes, not even possible. Therefore, in many cases, each subsystem was given its own Lyapunov-like function that behaved like Lyapunov function when that subsystem was active.

Interestingly, it was found that, even in the switched systems which did not possess a common Lyapunov function also exhibited asymptotically stable nature, under some properly chosen switching law. This observation found a loophole in the former notion of having a common Lyapunov function and led to the multiple Lyapunov function concept proposed by Peleties and DeCarlo (1991) which was later generalized by Branicky (1998). Multiple Lyapunov function techniques for ensuring stability of the switched systems are highlighted in [4-14] and many recent results in this direction are available in [15].

Several other approaches to the concept of stability have been defined in [16, 17, 18], including uniform stability, asymptotic, and exponential for switched systems. For instance, arbitrary switching problem can be tackled by construction of common quadratic and non-quadratic Lyapunov functions ensuring exponential stability. It is to be noted, that the converse may not be true, that is, the uniform exponential stability of systems under arbitrary switching does not mean that a CQLF (common quadratic Lyapunov Function) exists for its constituent systems [18]. Brayton and Tong [19], established equivalence of existence of a common Lyapunov function for constituent systems of a discrete time switched linear system and there is uniform stability of system under arbitrary switching.

Any system is uniformly exponentially stable for arbitrary switching signals, if a strictly convex, positive definite function V($\boldsymbol{x}$) exists, homogeneous of degree 2, of the following form:

$$V(\boldsymbol{x}) = \boldsymbol{x}^T L(\boldsymbol{x})\boldsymbol{x}, \tag{3}$$

where $L(\boldsymbol{x}) \in R^{n\times n}$, *and*

$$L(\boldsymbol{x})^T = L(\boldsymbol{x}) = L(c\boldsymbol{x}), \tag{4}$$

for all non-zero $c \in R, x \in R^n$ *such that* $max_{\boldsymbol{y} \in A\boldsymbol{x}} \frac{\delta V(x)}{\delta y} \leq -\gamma \|x^2\|$ *for some* $\gamma > 0$, *where* $A\boldsymbol{x}=\{A_1\boldsymbol{x}, \dots, A_m\boldsymbol{x}\}$ *and*

$$\frac{\delta V(x)}{\delta y} = inf_{t>0} \frac{V(x+ty)-V(x)}{t} \tag{5}$$

is the usual directional derivative of the convex function V(x) [20].

An effective method is to test the existence of a CQLF by using LMIs (Linear Matrix Inequalities). However, it has certain disadvantages as it does not prove as to why a CQLF may or may not exist for a set of LTI systems and further, LMI's have limitations in applicability when the sub-systems are large in number [20].

It is proved by Feron in 1996 that when number of sub systems is two, necessary and sufficient condition for quadratic stabilizability of switched system is the existence of a stable convex combination of the sub-systems. In 2001, Zhai extended the results to discrete-time switched linear systems, by giving non-negative combination of subsystems' Lyapunov inequalities as quadratic stabilizability condition.

The quadratic stability for class of switched nonlinear subsystems was given by Zhao and Dimirovski [21] by converting it to a nonlinear programming problem and consequently deriving necessary and sufficient condition using Karush–Kuhn–Tucker (KKT) condition.

Another basic tool to analyze switched systems is LaSalle's invariance principle [22, 23]. Assuming a common or multiple Lyapunov functions which decrease along all trajectories with a constraint on switching rate, the trajectories of a switched system approach an invariant set. These aspects are proved by Bacciotti, Hespanha and Mancilla-Aguilar [24, 25, 26]. Recent works include that of Lee [27] and by Liu [28] on arbitrary and impulsive switching, respectively.

Lie-algebraic stability criteria for switched systems are formulated for the original data. However, their main disadvantage is their limited applicability as they only provide sufficient but no necessary conditions for stability. Lie-algebraic stability criteria has been investigated by Agrachev and Liberzon [29] and stability of switched systems using Lie algebra is derived by Liberzon et al. [30,31] and Agrachev [32].

Considering the case of arbitrary switching problem, most results are based on existence of CQLFs. An asymptotically stable switched linear system for arbitrary switching sequences in which sub-systems don't have a CQLF, is not difficult to construct. Extensive research is being conducted to determine conditions of existence of non-quadratic Lyapunov functions. The converse theorem given by Molchanov and Pyatnitski's for arbitrary switching, states that underlying switched linear system is asymptotically stable for a common piecewise quadratic or a linear Lyapunov function. Works of Rosenbrock [33] and Weissenberger [34] on Lur'e-type systems on piecewise linear Lyapunov functions' existence, received acclaim in 1960s.

Usually, systems in the real world are affected by the external interference, such as disturbance in control or errors on observation. Time delay usually is inevitable in dynamic systems due to the external environment or internal factors, such as parameter variability, measurement, transmission or transport lags, and computational delays [35–46]. The problem of time delays in hybrid switched systems with interaction between the continuous dynamics and discrete switching is more difficult to study. Sun in his pioneering work [47] presented stability analysis of switched system with time-delay. He worked on stability problem for switched systems with dwell time and assumed a constraint on derivative of time-varying delay. The stability of some slow switched control systems is studied [48-52].

The entire system is said to be exponentially stable for any switching signal if all subsystem matrices are Hurwitz stable (i.e., all eigenvalues lie in left half complex plane), considering time between consecutive switching's (dwell time) is sufficiently large. In case of linear switched systems with both Hurwitz stable and unstable subsystems and considering average dwell time to be sufficiently large along with total activation time of the unstable subsystems to be relatively small compared with that of the Hurwitz stable subsystems, then global exponential stability is ensured [53-55].

External disturbance inputs and time delay often lead to loss of stability for an otherwise stable system [39–46]. In these circumstances the input-to-state stability (ISS) must be guaranteed. The notion of ISS was first introduced by Sontag [56]. ISS means that if the external input is small, then no matter the initial state, the state must be eventually small and has proven useful in designing controllers for nonlinear systems [56-61].

Another powerful control method to counteract external disturbances is sliding mode control technique [62]. The switching pattern in which switching times form an infinite sequence accumulating near final time is known as Fuller's phenomenon, or Zeno behavior and is fully understood from optimal control theory [63, 64]. Such controllers are called chattering controllers [64]. However, they can provide the desired system performance despite of significant uncertainties in system. Suboptimal chattering control algorithms, also known as second order sliding mode control algorithms, have subsequently been developed for SISO (Single Input-Single Output) systems [65] to guarantee asymptotic stability of closed-loop system, while retaining useful robustness features against matching disturbances.

Turning to L2-gain analysis and H_∞ control problems, they have been studied in relation to algebraic Riccati inequalities for linear systems and Hamilton–Jacobi inequalities for nonlinear systems [66]. H_∞ control for uncertain discrete switched systems was studied by Lin and Antsaklis in 2003.

As mentioned before, interesting behavior can be exhibited, under appropriate switching laws by switched systems whose subsystems have no equilibria but behave like asymptotically stable systems near an equilibrium. This can be described as practical stabilizability (local behavior) and practical asymptotic stabilizability (behavior in a larger region) [67-71]. Their goal is to bring the system trajectories within the given bounds [72, 73]. Since several reported switching strategies cause undesirable Zenoness, therefore, efforts have been made to construct negative switching sequences by incorporating dwell-time or hysteresis in the switching laws [74, 75]. However, it is often the case that by introducing such modified switching laws, it is required that asymptotic stabilizability of equilibrium is compromised for asymptotic convergence to a neighborhood around equilibrium.

III. Controllability Aspects of Switched Systems

Controllability is a dynamical property which is inseparably linked to the random events that might occur, like system failures or other accidental disturbances [76-79]. Controllability implies the capability to move a system around its entire configuration space using only certain admissible controls. Controllability oncept for both time-invariant and time-varying linear control systems is based on state space description. Kalman was first to propose it in [80] for linear dynamical systems.

Stiver and Antsaklis discussed the controllability of supervisory hybrid systems [81] using controllable language concept, derived from Discrete Event Systems theory [82]. Tittus and Egardt defined 'hybrid controllability' employing hybrid automation system as system model [83]. Consistent with hybrid controllability, Caines and Wei proposed the 'between-block controllability' [84]. Related definitions can be found in the work highlighted [84-89].

In literature, the following definitions have been introduced depending on the type of models applied: approximate [90], complete [91], asymptotic [92], exact [93], constrained [94], null [95, 96], global [97], output [98-100] and relative controllability [101]. Various controllability problems for different systems have been considered in the work presented in [102-105].

For more general hybrid control systems, sufficient condition for hybrid controllability was given by Schuppen [87]. Lemch discussed global controllability of autonomous switched hybrid systems and obtained sufficient conditions in terms of 'hybrid fountains' [85]. The necessary and sufficient conditions were derived by Xie, Wang and Yang [88, 89] for switched linear systems.

Recent study by Liu, Lin, & Chen [106] on controllability was done for a class of uncertain switched linear systems in which the state matrices are either unknown or zero. They developed the concept which became useful in scenarios where system parameters were difficult to trace and were obtained with some approximation error. If unknown parameters values are so that corresponding switched linear system is controllable and system becomes structurally controllable.

IV. Observability of Switched Systems

The concept of observability is documented in the classical linear theory [107], and has gained wide attention. However, in the switched case it gets very complicated as the discrete modes of the switched systems may or may not be observed. Observability and controllability are closely related to pole assignment, structural decomposition and quadratic optimal control etc. One or multiple-period controllability, and observability were first studied for periodically switched systems in the study by Ezzine and Xie [108] and [109], respectively. They concluded that controllability could only be realized in n periods, where n is state dimension [110].

It is to be noted that switching phenomena can occur due to active switching as well as component failures. Mode observability allows the detection of such failures by recovering initial states and switching signal from the output. This was studied by Babaali and Pappas in 2005 and by Elhamifar in 2009. Observability of linear hybrid systems is studied in [111-115], where the modes are considered to be dependent on state trajectory. Deterministic discrete-time switched systems are documented by Babaali and Vidal [116,117].

There are numerous ways to define observability for linear time-invariant systems with output. 'Distinguishability' states that different initial conditions produce different outputs. This property is equivalent to 0-distinguishability (negative initial conditions produce nonzero outputs). Thus, the state of an observable linear system can be reconstructed from output measurements by inverting the observability Gramian on a time interval of arbitrary length. However, in nonlinear context definitions of observability are not equivalent [118-120].

'Detectability' is another concept related to observability. A variant of detectability is 'output-to-state stability' where when inputs and outputs are zero, the states should converge to zero, and in general be ultimately bounded by magnitude of inputs and outputs. The following should hold true:

$$|x(t,\xi)| \leq \max\{\, \beta\,(|\xi|,t)\,,\gamma_1\,(\|u_{\{0,t\}}\|)\,,\gamma_2\,(\|y_{\{0,t\}}\|)\} \tag{6}$$

For every initial state ξ. Here $|\xi|$ represents Euclidean norm, $\|u_{\{0,t\}}\|$ and $\|y_{\{0,t\}}\|$ denotes sup norms of input and output y = h(x(t)) respectively, x(t) being the solution with x(0)= ξ and input u(.) on interval [0,t], functions γ_i are of class K, that is zero at zero, strictly increasing, and continuous, and β is a function of class KL, i.e. it decreases to zero on t and is of class K on **x** [121]. At times, instead of building an observer, it is sufficient to construct a "norm estimator" to obtain an upper bound on the norm of the state using the output [121].

V. Important Issues of Hybrid Dynamical Systems

Following are the basic difficulties during analysis of even simple hybrid dynamical systems:

- Arbitrary switching- It deals with the question of common Lyapunov function existence. Converse theorems prove existence of common Lyapunov functions under assumption of exponential stability.
- Dwell-time- the problem to determine minimum length of time that must elapsed between successive switches to ensure stability of system. Some of the recent research have been reported in [122] for neutral stochastic switched time delay systems, whose dynamics depends not only on the past and present states but also on its derivatives with delays and are subjected to environmental disturbances. [123] Addresses one of the most fundamental problem in nonlinear control, that is of global output feedback stabilization of switched nonlinear systems as separation principle no longer holds. Singular switched systems have gained popularity in the recent past [123-129]. Their global stabilization via dwell time approach is studied in [130].
- Stabilization- We know that switching between stable systems can create instability, similarly a family of individually unstable systems can be stabilized by appropriately switching between them. Since then, several studies have been undertaken to find a solution to problem of determining such stabilizing switching laws [131, 132].
- Chaos- Chase, Serrano, and Ramadge worked out how chaotic behavior arises while switching between low-dimensional linear vector fields [133].
- Complexity- questions regarding the complexity and decidability of stability of switched systems [134,135],

and precise nature of connection between stability under arbitrary switching and stability under periodic switching rules (periodic stability) are important aspects which are needed to be explored in depth [136-140].

VI. Future Directions

Though extensive research is being conducted in the domain of switched systems, however, many problems still remain, such as; determining the stabilizing switching signals, the problem of dwell-time, and the stability under arbitrary switching.

It is interesting to determine system classes, for instance, class of second order systems or pairs of systems with system matrices differing by a rank one matrix, for which simple conditions for CQLF existence can be given. One such noteworthy work is reported by Shorten et al. [141].

A closely related problem is to determine classes of switched linear systems under arbitrary switching, where CQLF existence is equivalent to exponential stability.

For the class of positive switched linear systems, where constraint is nonnegative orthant, Lyapunov functions may lead to less conservative stability criteria than those obtained through requiring CQLF existence. Thus, problem raised is to determine verifiable conditions for common co-positive Lyapunov function existence for families of positive LTI systems.

Other than these issues of stability for arbitrary switching signals, another unresolved high priority issue is to determine non-conservative estimates of the dwell-time for constrained switching.

To determine the stabilizing switching signals for unstable constituent systems, work of Feron et al. [142] provides sufficient conditions for quadratic stabilization laws. However, a query remains unsolved about the necessary and sufficient conditions for existence of general stabilizing switching laws.

VII. Conclusion

This paper has studied the development in the field of switched systems and underlying methodologies to ensure stabilization of systems with stable or unstable dynamics. Controllability and observability aspects of these systems are highlighted which work as basis for stability exploration of such systems. Noted contributions in the area of switched discrete/continuous time systems and hybrid systems are summarized for quick reference. Further, current status and future directions of possible scope and open challenges in this area are also highlighted for completeness.

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